Prospects for real quantum information processing devices in the laboratory

David DiVincenzo, IBM

Computational Sciences Lecture Series,
UW Madison, 2/11/05

-- implementations – criteria & possibilities
--many qubits, not working so well
-- Josephson circuits: where's the qubit?
--circuit mechanics theory

Work with:
Roger Koch and company
Guido Burkard
Fred Brito

Five criteria for physical implementation of a quantum computer

1. Well defined extendible qubit array -stable memory
2. Preparable in the "000…" state
3. Long decoherence time (>10^4 operation time)
4. Universal set of gate operations
5. Single-quantum measurements

D. P. DiVincenzo, in Mesoscopic Electron Transport, eds. Sohn, Kowenhoven, Schoen (Kluwer 1997), p. 657, cond-mat/9612126; "The Physical Implementation of Quantum Computation," Fort. der Physik 48, 771 (2000), quant-ph/0002077.

Five criteria for physical implementation of a quantum computer & quantum communications

1. Well defined extendible qubit array -stable memory
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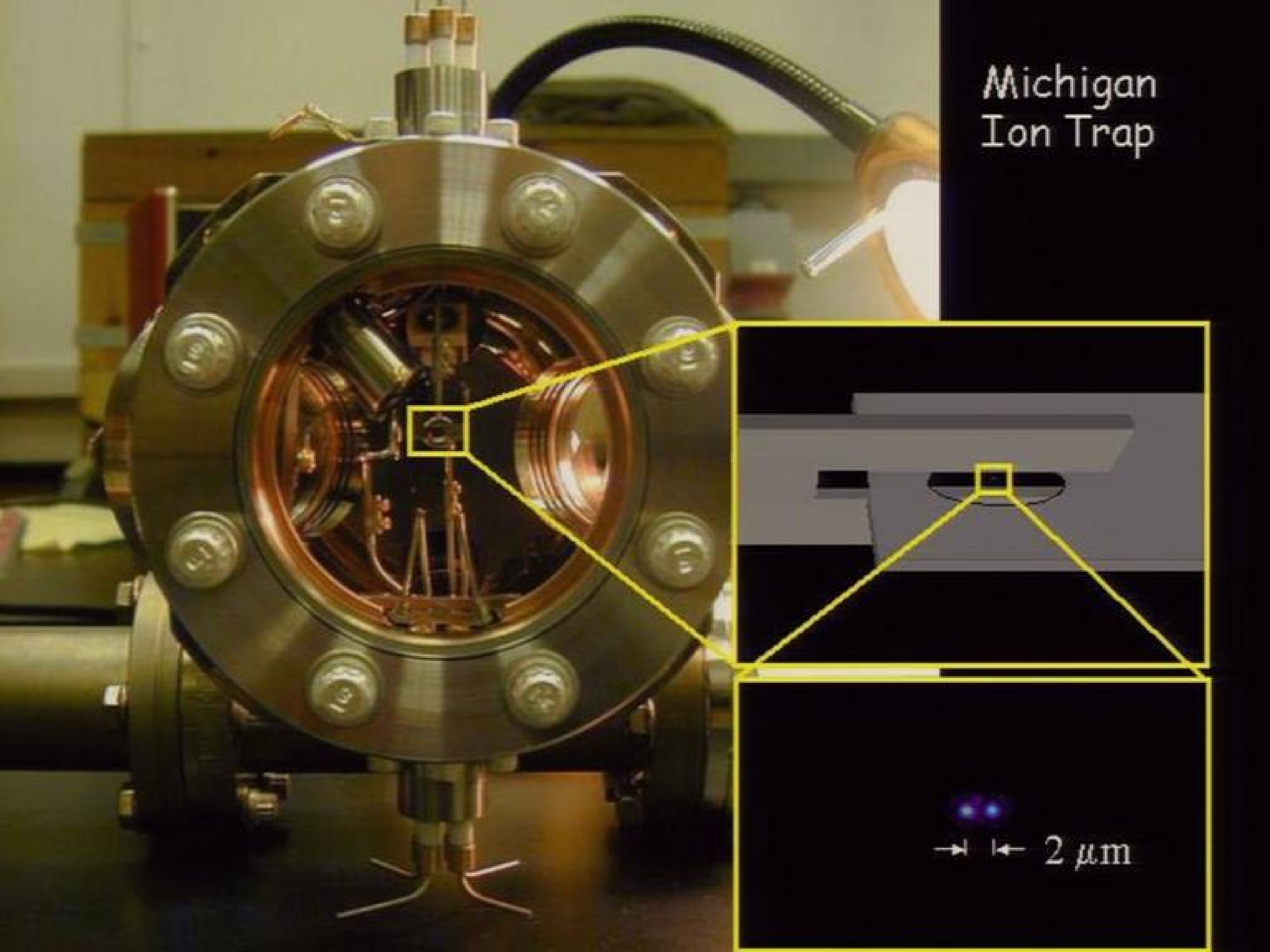
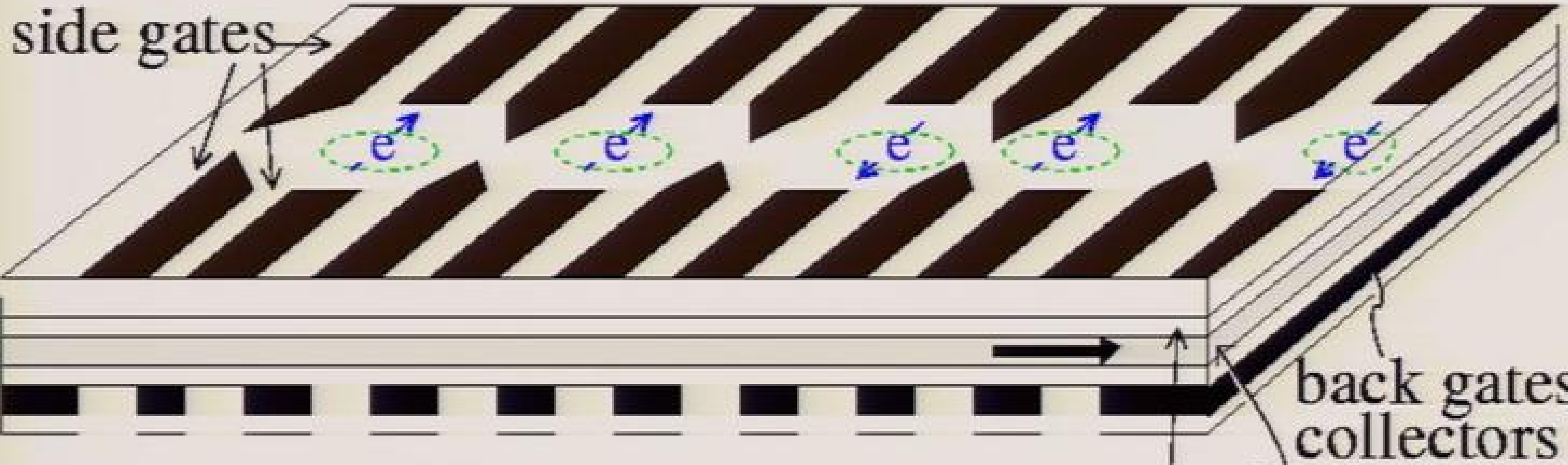
3. Long decoherence time (>10^4 operation time)
4. Universal set of gate operations
5. Single-quantum measurements
6. Interconvert stationary and flying qubits
7. Transmit flying qubits from place to place


Michigan
Ion Trap
2 μm

Quantum–dot array proposal:

Loss & DiVincenzo, Phys. Rev. A 57, 120 (1998).



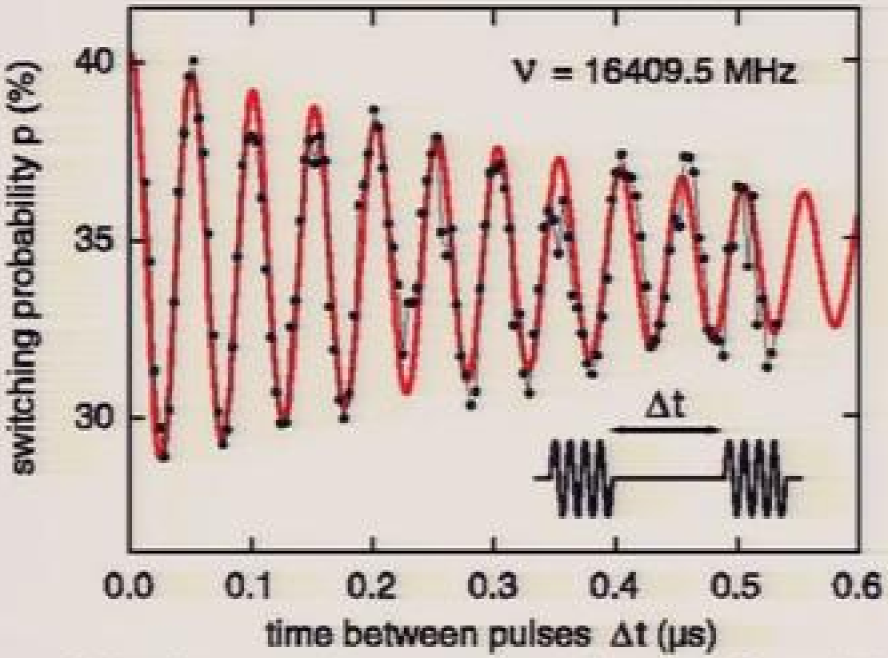
–quantum dots defined in 2DEG by side gates
–Coulomb blockade used to fix electron number at one per dot
–spin of electron is qubit
–gate operations: controllable coupling of dots by point–contact gate voltage
–readout by gatable magnetic barrier

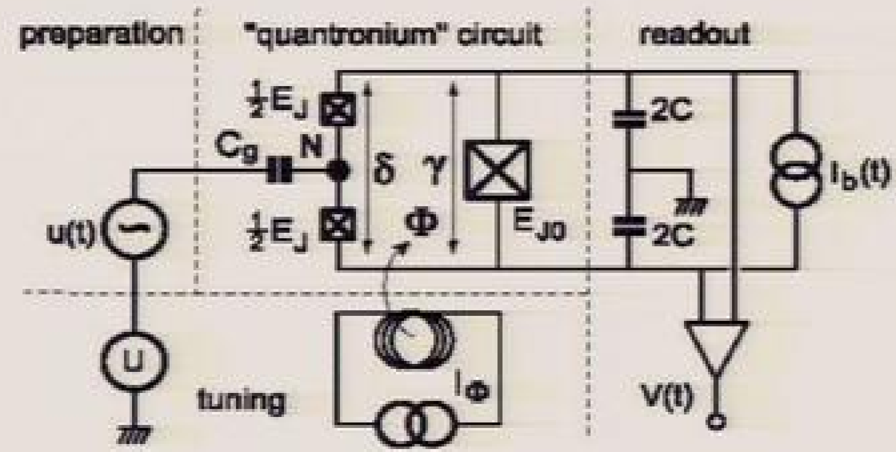
Josephson junction qubit -- Saclay

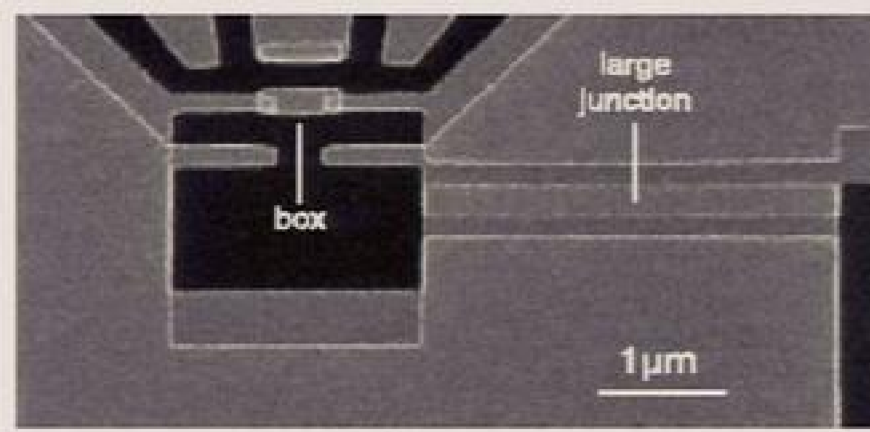
Manipulating the quantum state of an electrical circuit

Science 296, 886 (2002)

D. Vion, A. Aassime, A. Cottet, P. Joyez, H. Pothier, C. Urbina, D. Esteve and M.H. Devoret



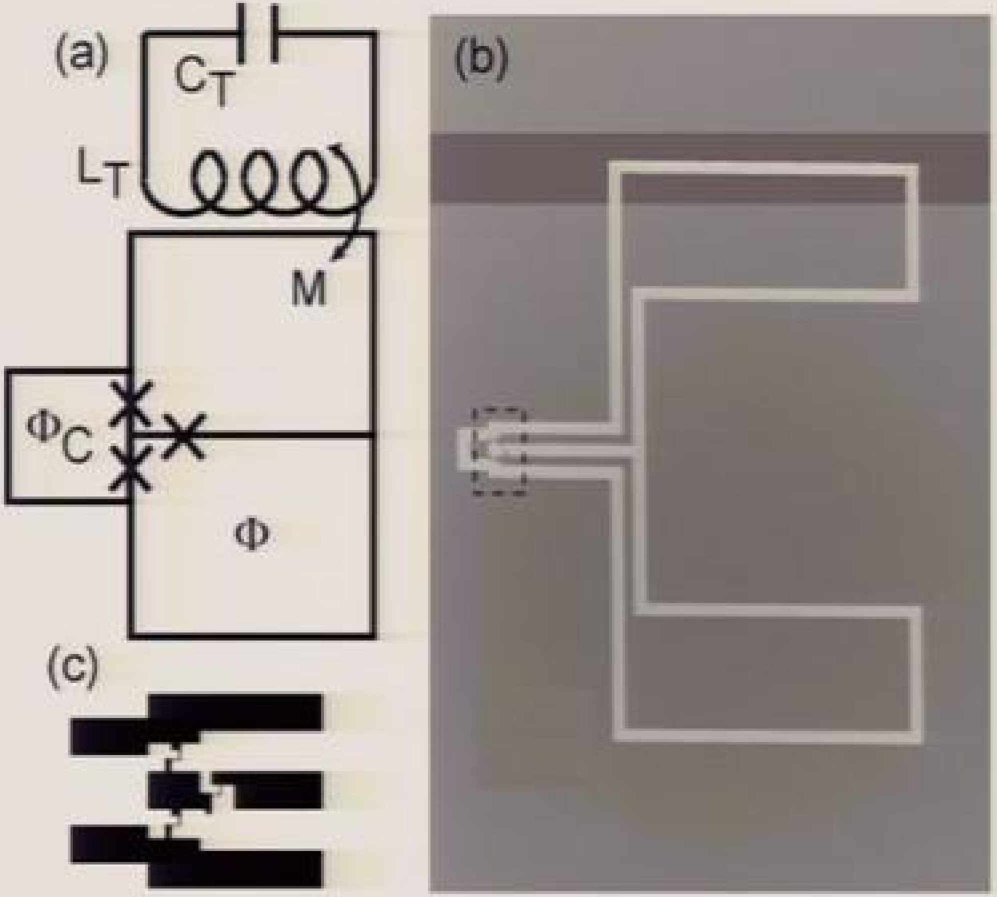




Oscillations show rotation of qubit at constant rate, with noise.

Where's the qubit?

IBM Josephson junction qubit

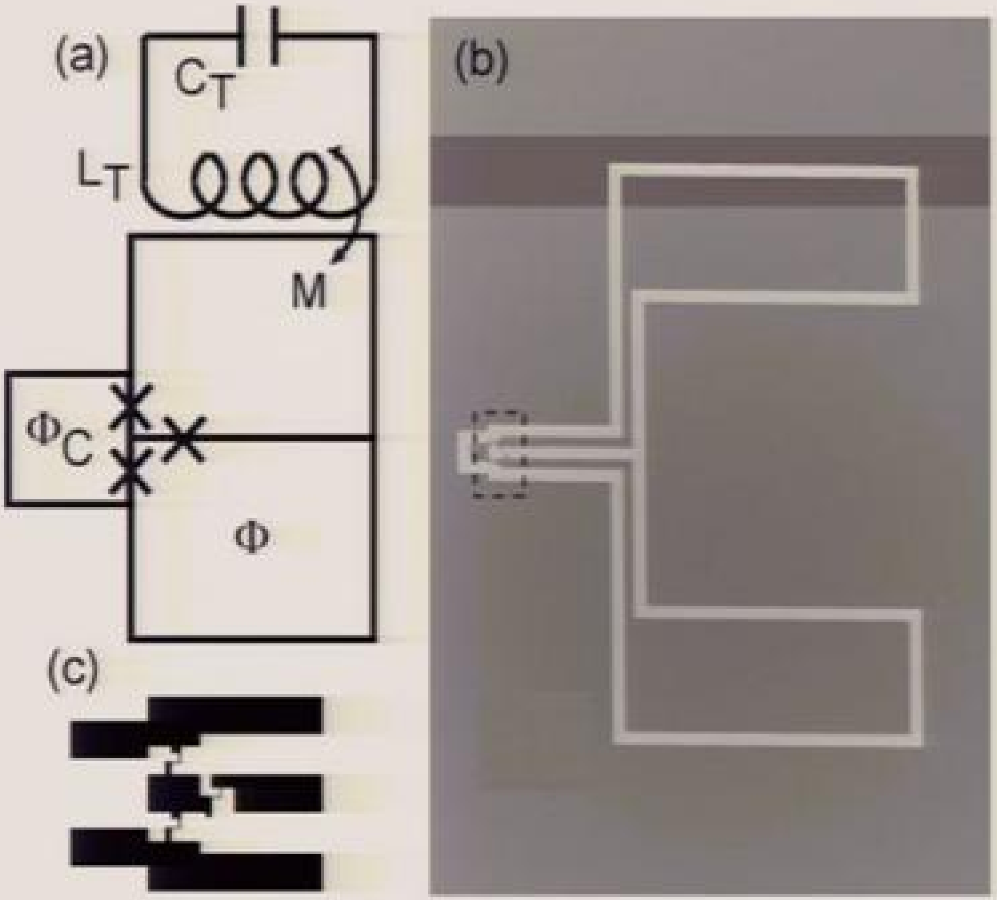


“qubit = circulation of electric current in one direction or another (????)

Low-bandwidth control scheme for an oscillator stabilized Josephson qubit

R. H. Koch, J. R. Rozen, G. A. Keefe, F. M. Milliken, C. C. Tsuei, J. R. Kirtley, and D. P. DiVincenzo
IBM Watson Research Ctr., Yorktown Heights, NY 10598 USA
(Dated: November 16, 2004)

IBM Josephson junction qubit



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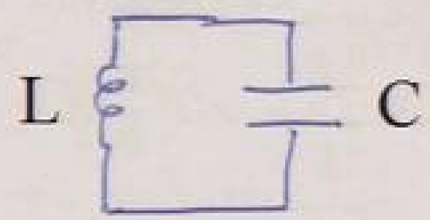
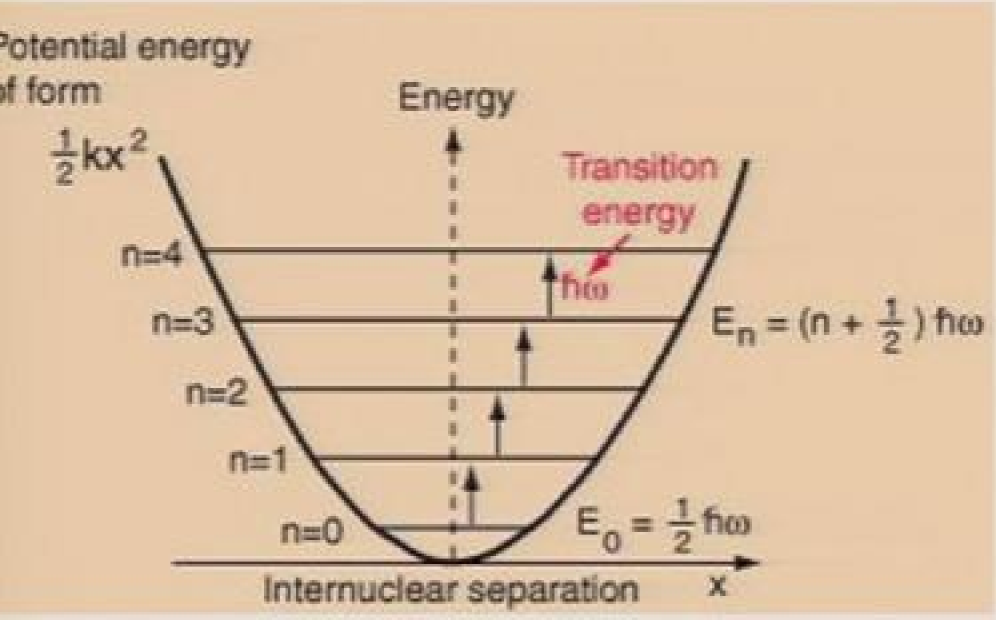
Understanding systematically the quantum description of such an electric circuit…

Simple electric circuit…



harmonic oscillator with resonant frequency $\omega_0 = 1/\sqrt{LC}$

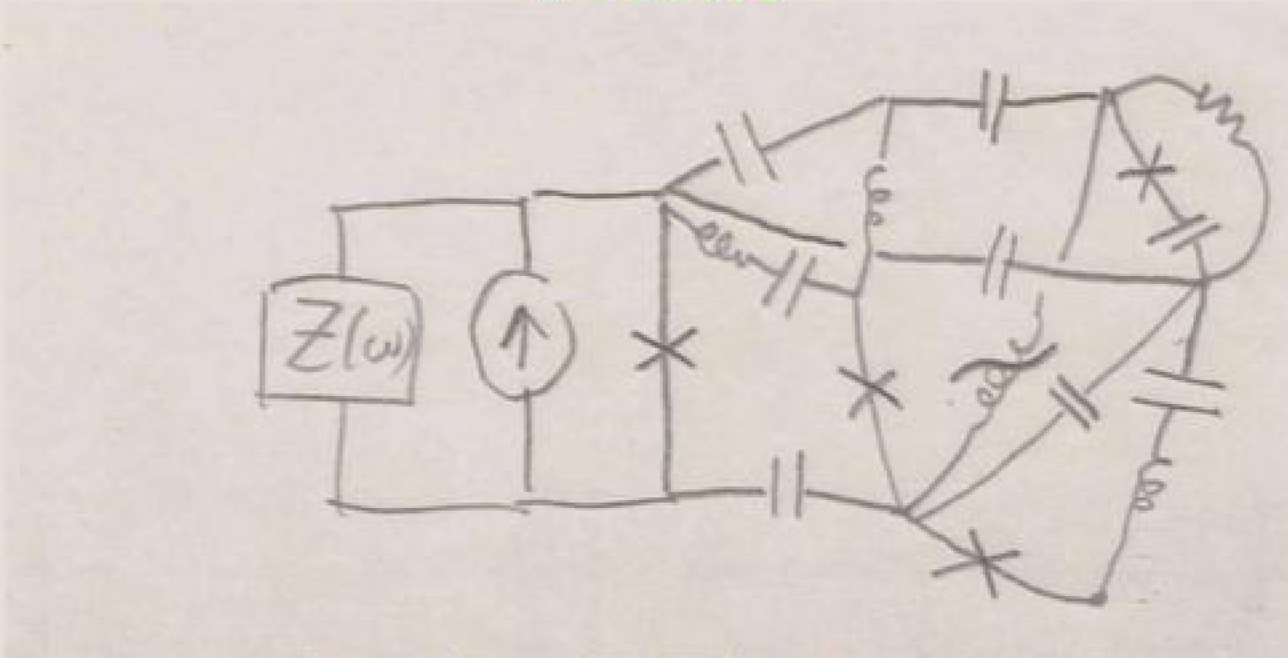
Quantum mechanically, like a kind of atom (with harmonic potential):



x is any circuit variable (capacitor charge/current/voltage, Inductor flux/current/voltage)

That is to say, it is a "macroscopic" variable that is being quantized.

But we will need to learn to deal with…



--Josephson junctions
--current sources
--resistances and impedances
--mutual inductances
--non-linear circuit elements?

G. Burkard, R. H. Koch, and D. P. DiVincenzo, "Multi-level quantum description of decoherence in superconducting flux qubits," Phys. Rev. B **69**, 064503 (2004); cond-mat/0308025.

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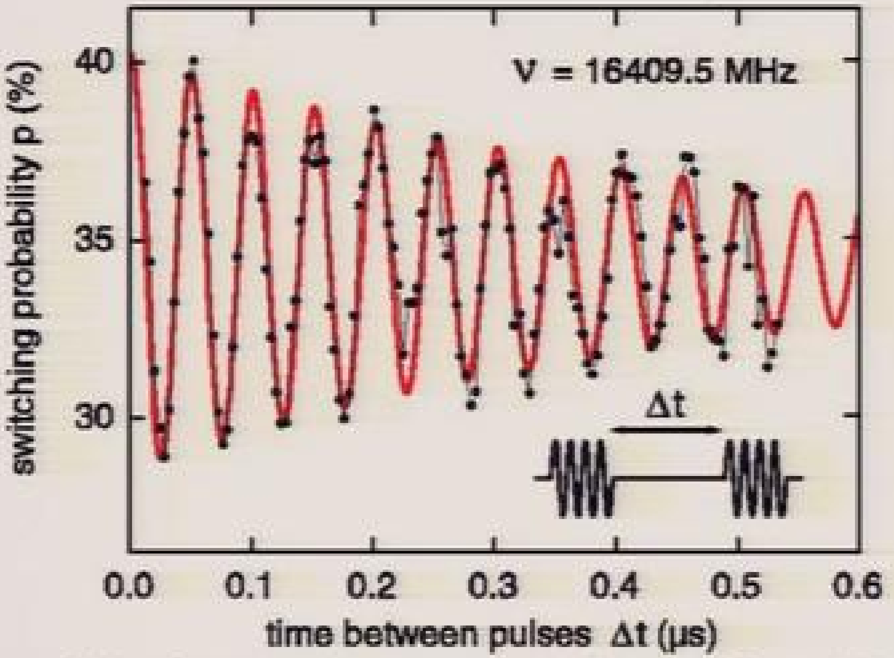

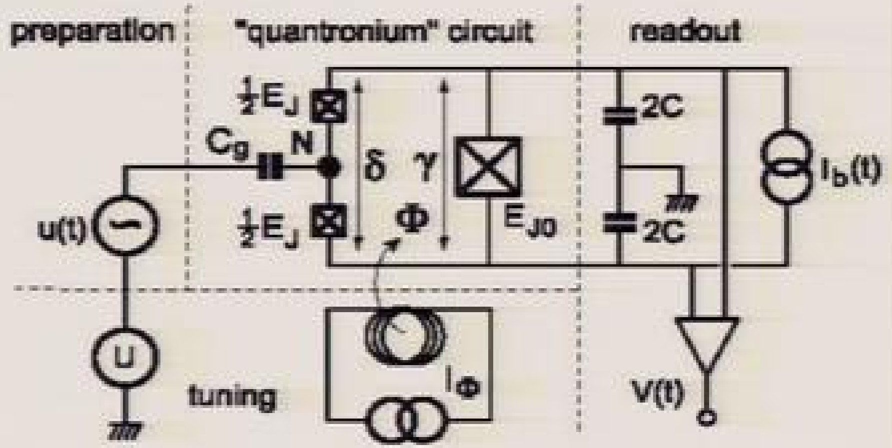

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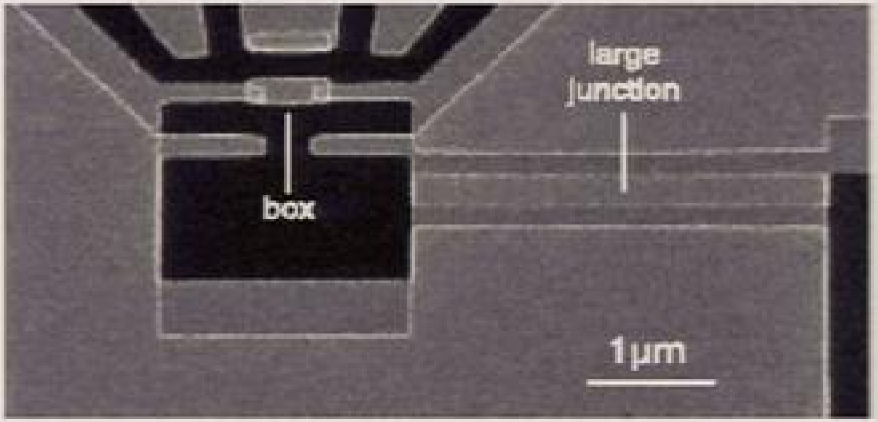
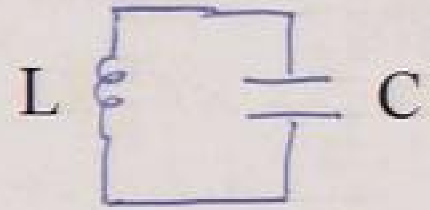


Where's the qubit?

Simple electric circuit…



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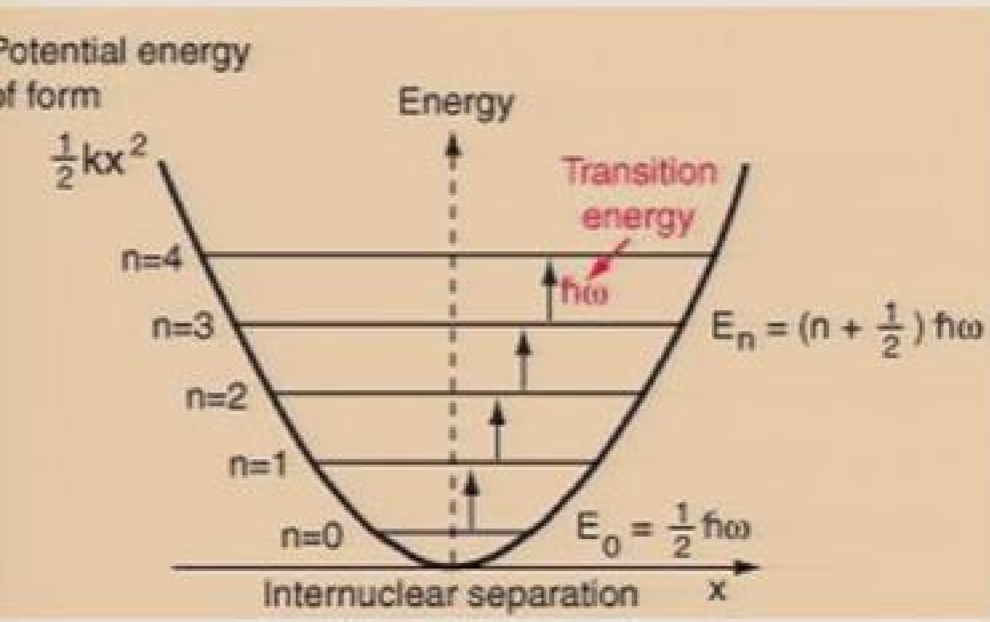

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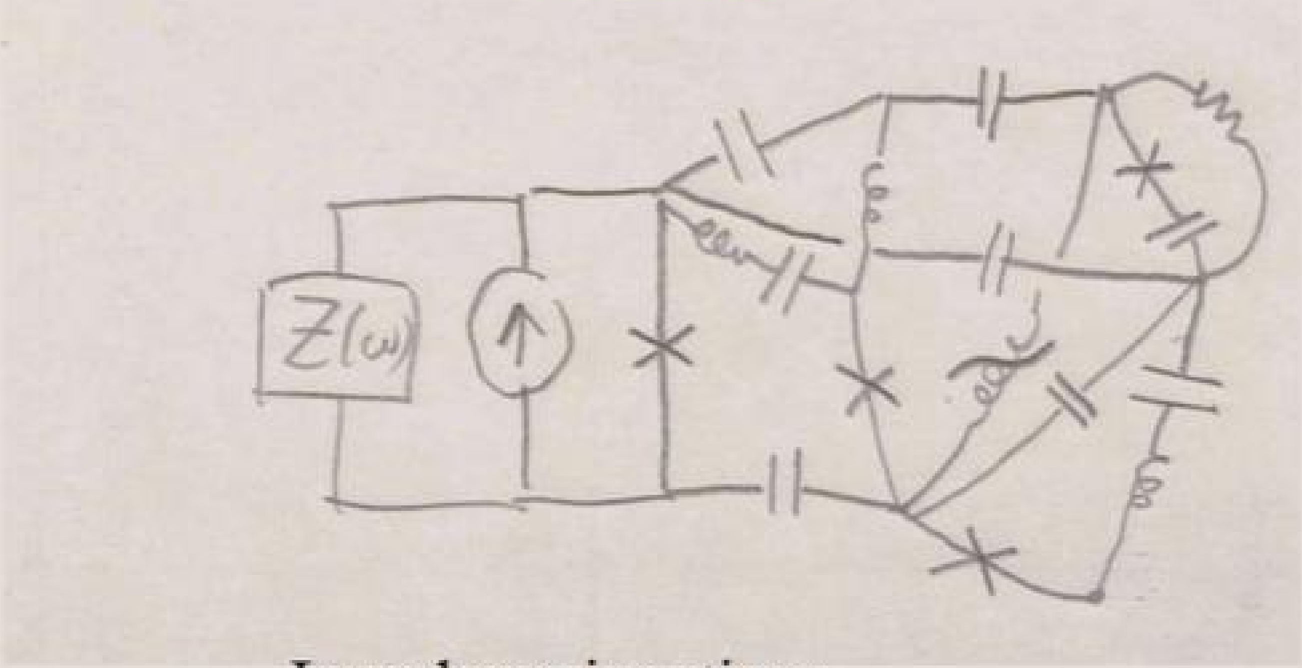

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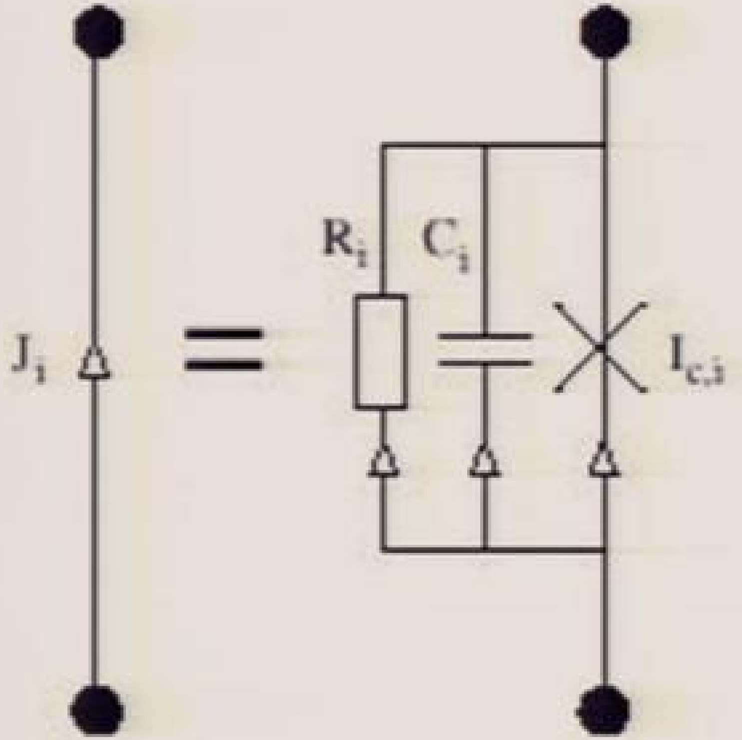
Josephson junction circuits

Practical Josephson junction is a combination of three electrical elements:



Ideal Josephson junction (x in circuit): current controlled by difference in superconducting phase phi across the tunnel junction:

$$I_J = I_c \sin\varphi$$

Completely new electrical circuit element, right?

small

not really…

What's an inductor (linear or nonlinear)?

$$\Phi = LI, \quad \text{(instantaneous)}$$

$$I = L^{-1}\Phi$$

$$I = L^{-1}(\Phi)$$

Φ is the magnetic flux produced by the inductor

$$\dot{\Phi} = V$$

(Faraday)

Ideal Josephson junction:

$$\mathrm{I}_J = \mathrm{I_c} \sin\varphi$$

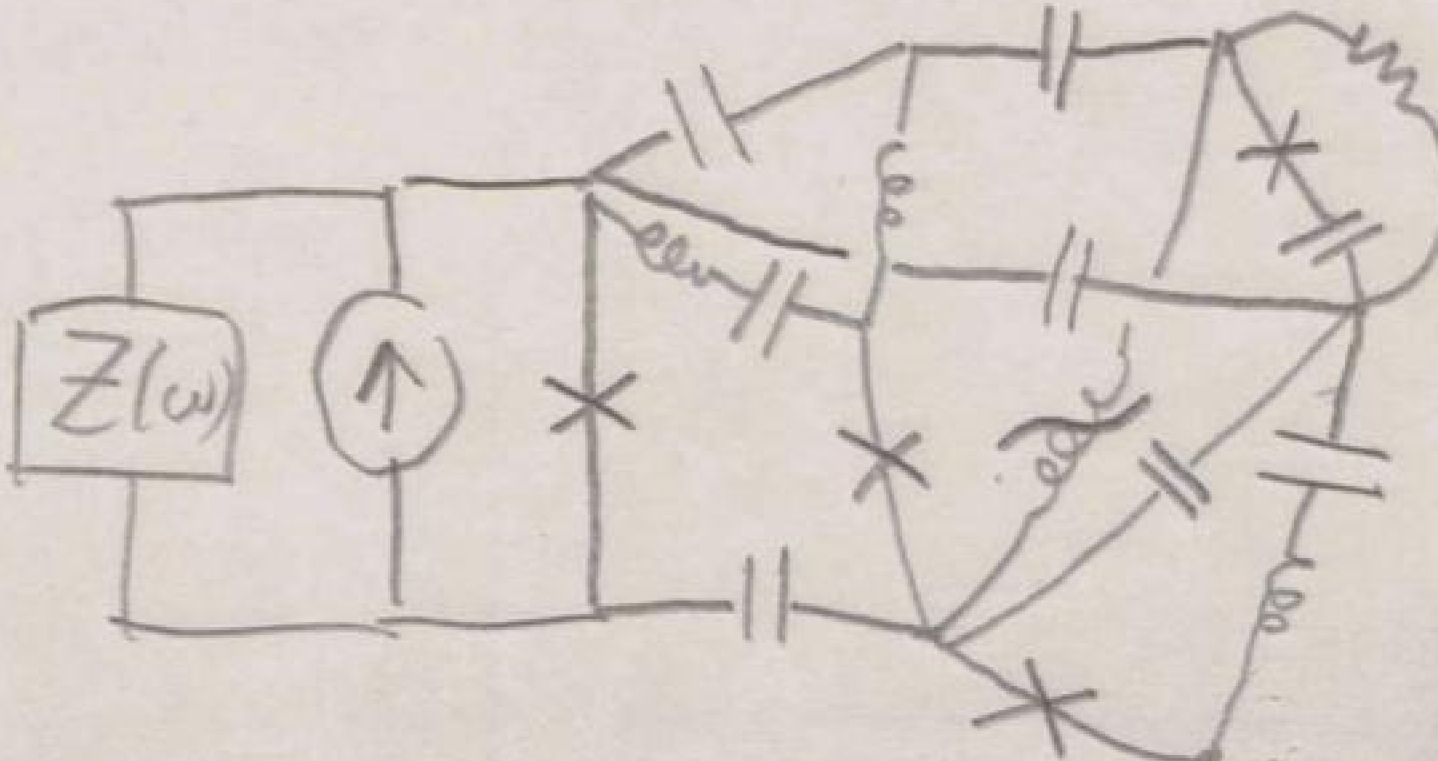
φ is the superconducting phase difference across the barrier

$$\frac{\Phi_0}{2\pi}\dot{\varphi} = V$$

(Josephson's second law)

$$\Phi_0 = h/e \quad \text{flux quantum}$$

So, we now do the systematic quantum theory



Graph formalism

1. Identify a "tree" of the graph – maximal subgraph containing all nodes and no loops

Branches not in tree are called "*chords*"; each chord completes a *loop*

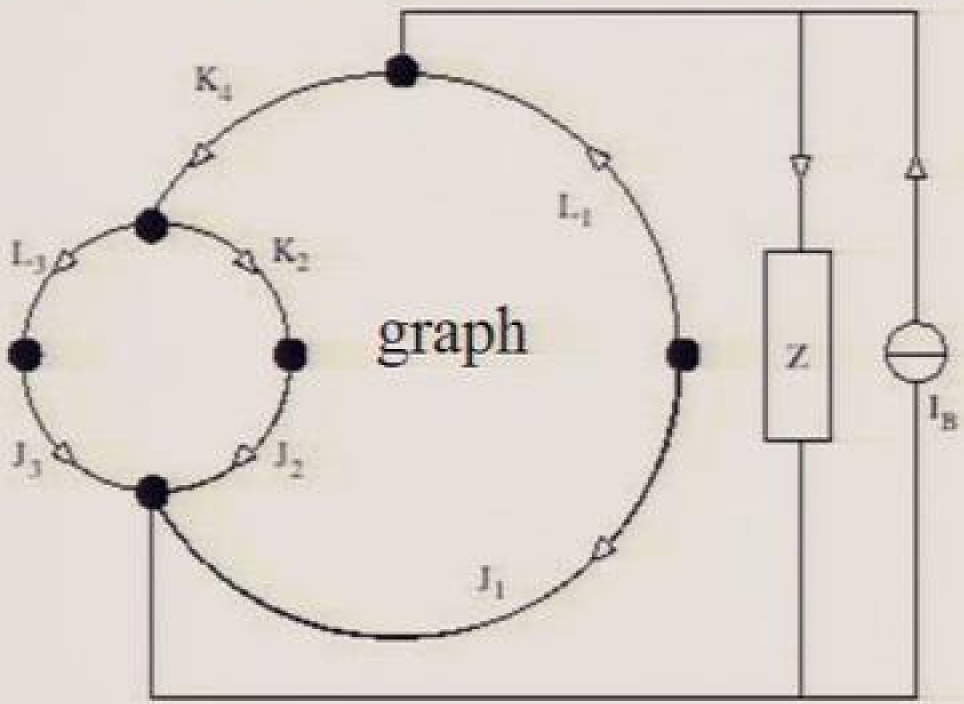
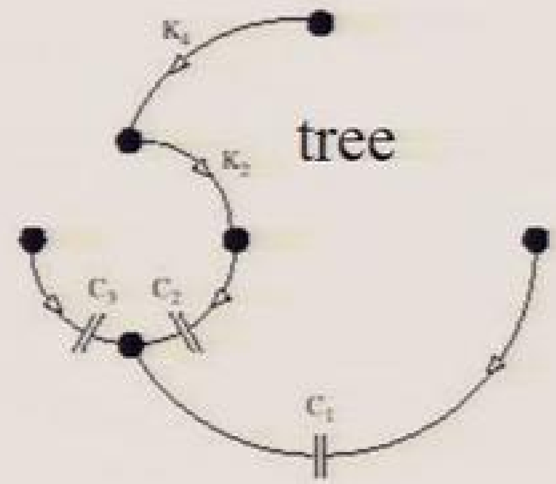


FIG. 1: The IBM qubit. This is an example of a network graph with 6 nodes and 15 branches. Each thick line represents a Josephson element, i.e. three branches in parallel, see Figure 2. Thin lines represent simple two-terminal elements, such as linear inductors (L, K), external impedances (Z), and current sources (I_B).

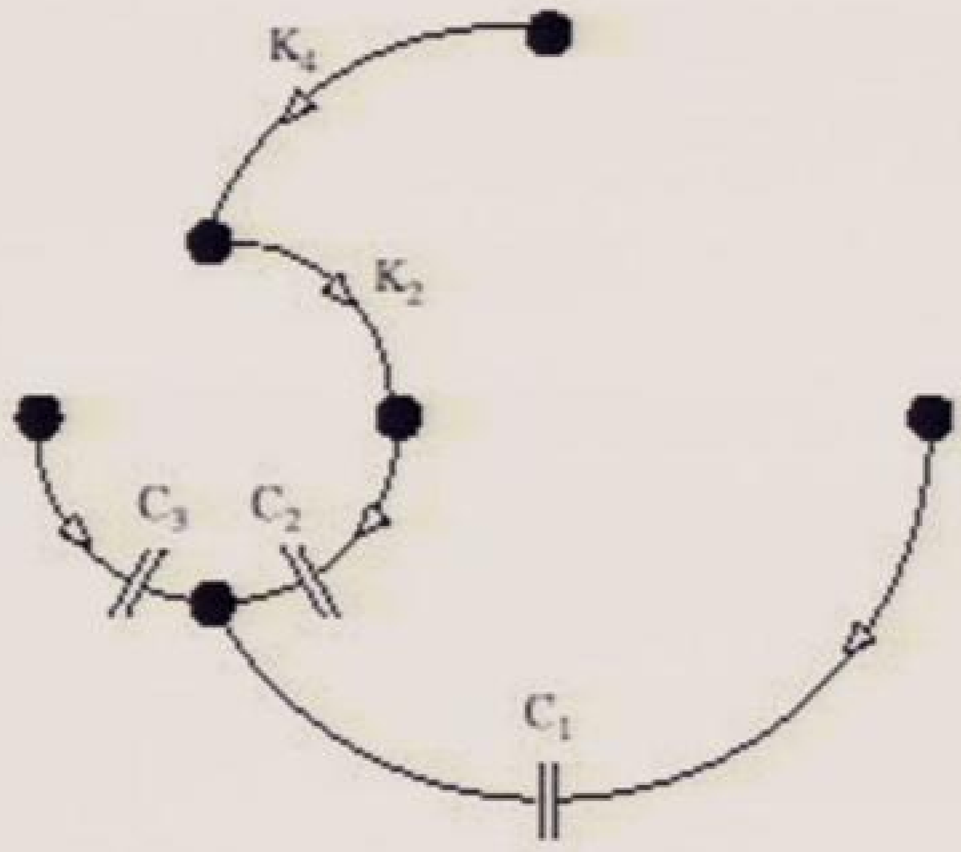


› for the circuit shown in Figure 1. A tree is a
aining all nodes and no loop. Here, we choose
ntains all capacitors (C), some inductors (K),
sources (I_B) or external impedances (Z).

graph formalism, continued

e.g., $$\mathbf{F}_{CL} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \\ 0 & -1 \end{pmatrix}$$

NB: this introduces submatix of F labeled by branch type



Circuit equations in the graph formalism:

Kirchhoff's current laws:

$$\mathbf{F}^{(C)}\mathbf{I} = 0$$

V: branch voltages
I: branch currents
Φ: external fluxes threading loops

Kirchhoff's voltage laws:

$$\mathbf{F}^{(L)}\mathbf{V} = \dot{\mathbf{\Phi}}$$

With all this, the equation of motion:

The tricky part: what are the independent degrees of freedom?

If there are no capacitor-only loops (i.e., every loop has an inductance),

then the independent variables are just the Josephson phases, and the "capacitor phases" (time integral of the voltage):

$$\mathbf{C}\ddot{\varphi} = -\mathbf{L}_J^{-1}\mathbf{sin}\varphi - \mathbf{R}^{-1}\dot{\varphi} - \mathbf{M}_0\varphi - \mathbf{M}_d * \varphi - \frac{2\pi}{\Phi_0}\mathbf{N}\mathbf{\Phi}_x - \frac{2\pi}{\Phi_0}\mathbf{S}\mathbf{I}_B$$

"just like" the biassed Josephson junction, except…

the equation of motion (continued):

$$\mathbf{C}\ddot{\boldsymbol{\varphi}} = -\mathbf{L}_J^{-1}\mathbf{sin}\boldsymbol{\varphi} - \mathbf{R}^{-1}\dot{\boldsymbol{\varphi}} - \mathbf{M}_0\boldsymbol{\varphi} - \mathbf{M}_d * \boldsymbol{\varphi} - \frac{2\pi}{\Phi_0}\mathbf{N}\boldsymbol{\Phi}_x - \frac{2\pi}{\Phi_0}\mathbf{S}\mathbf{I}_B$$

$$\begin{aligned}
\mathbf{M}_0 &= \mathbf{F}_{CL}\tilde{\mathbf{L}}_L^{-1}\bar{\mathbf{L}}\mathbf{L}_{LL}^{-1}\mathbf{F}_{CL}^T, \\
\mathbf{N} &= \mathbf{F}_{CL}\tilde{\mathbf{L}}_L^{-1}\bar{\mathbf{L}}\mathbf{L}_{LL}^{-1}, \\
\mathbf{M}_d(\omega) &= \bar{\mathbf{m}}\bar{\mathbf{L}}_Z^{-1}(\omega)\bar{\mathbf{m}}^T, \\
\bar{\mathbf{m}} &= \mathbf{F}_{CZ} - \mathbf{F}_{CL}(\mathbf{L}_{LL}^{-1})^T\bar{\mathbf{F}}_{KL}^T\tilde{\mathbf{L}}_K^T\mathbf{F}_{KZ} \\
\mathbf{S} &= \mathbf{F}_{CB} - \mathbf{F}_{CL}(\mathbf{L}_{LL}^{-1})^T\bar{\mathbf{F}}_{KL}^T\tilde{\mathbf{L}}_K^T\mathbf{F}_{KB}
\end{aligned}$$

All are complicated but straightforward functions of the topology (F matrices) and the inductance matrix

the equation of motion (continued):

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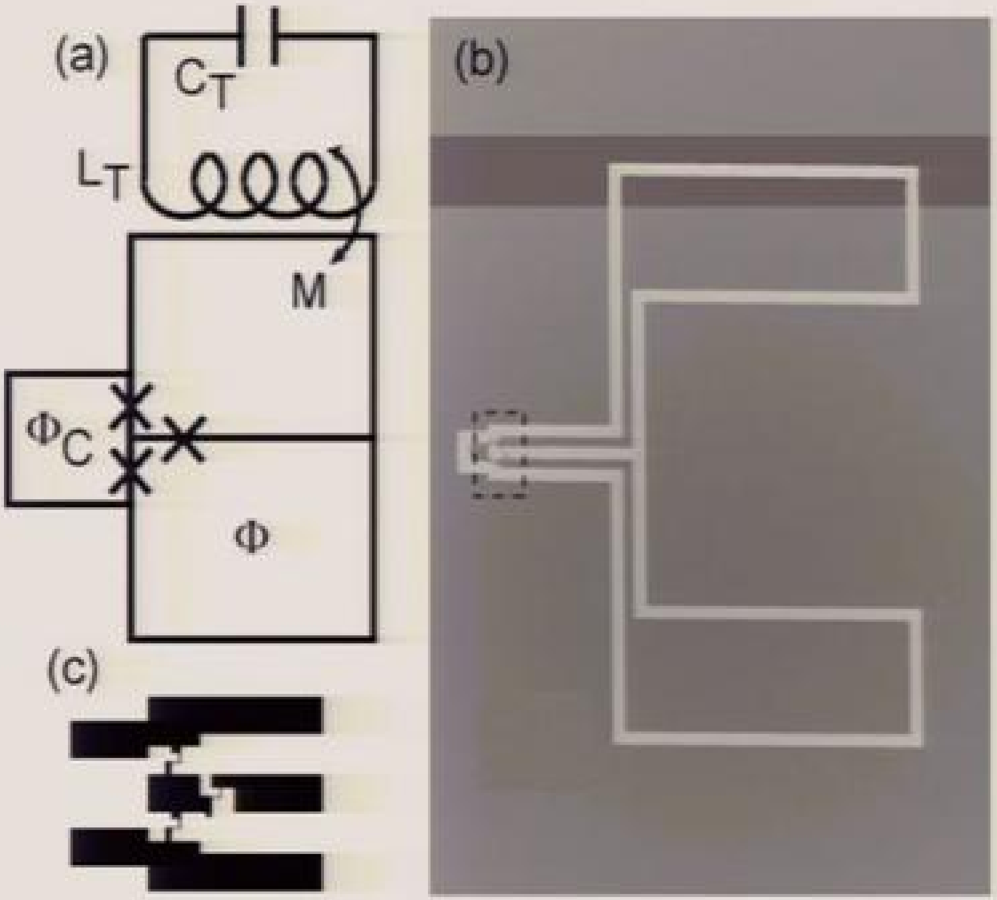
The lossless parts of this equation arise from a simple Hamiltonian:

H; U=exp(iHt)

$$\frac{1}{2}\mathbf{Q}_C^T\mathbf{C}^{-1}\mathbf{Q}_C + U(\boldsymbol{\varphi})$$

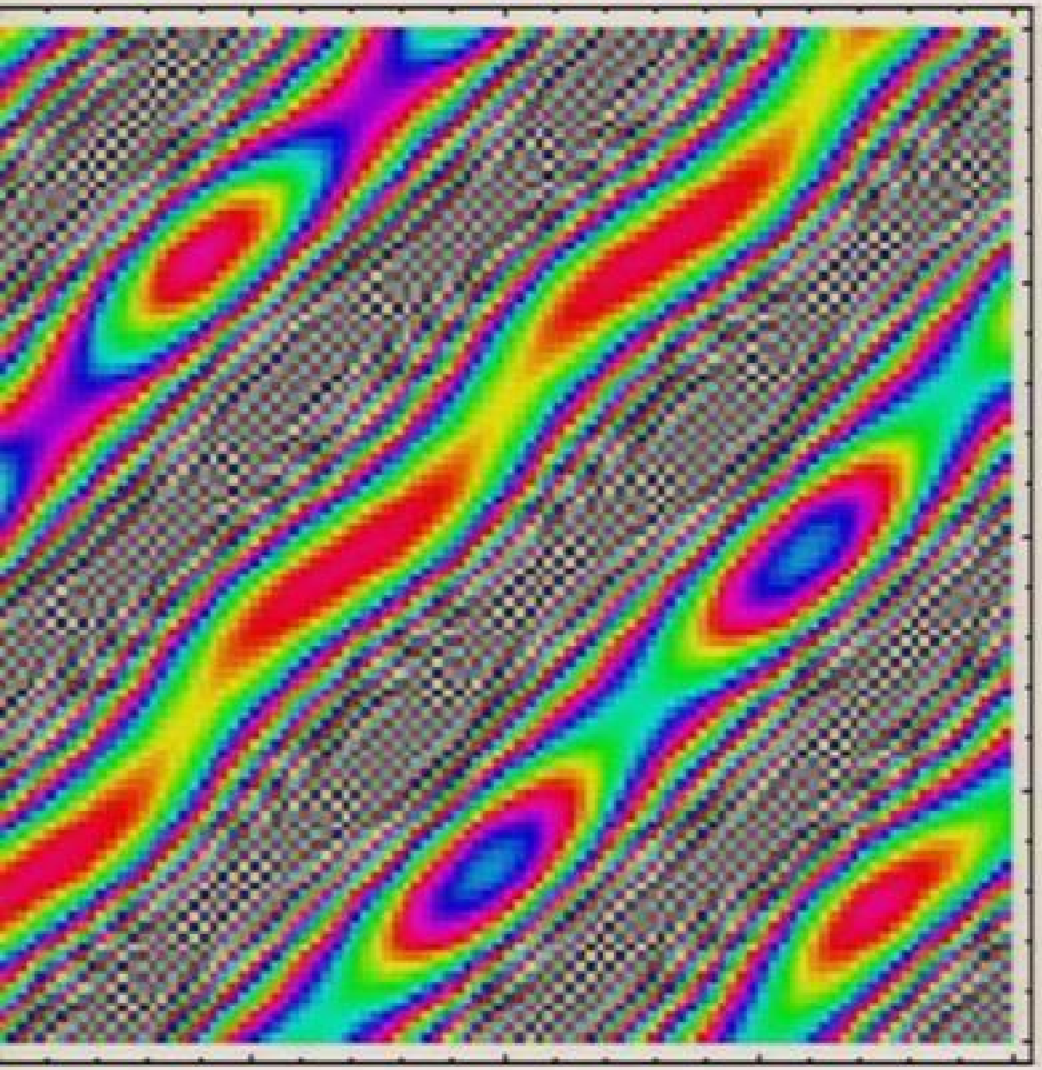
$$U(\boldsymbol{\varphi}) = -\sum_i L_{J;i}^{-1}\cos\varphi_i + \frac{1}{2}\boldsymbol{\varphi}^T\mathbf{M}_0\boldsymbol{\varphi} + \frac{2\pi}{\Phi_0}\boldsymbol{\varphi}^T\left(\mathbf{N}\boldsymbol{\Phi}_x + \mathbf{S}\mathbf{I}_B\right)$$

IBM Josephson junction qubit



Results for quantum potential of the gradiometer qubit…

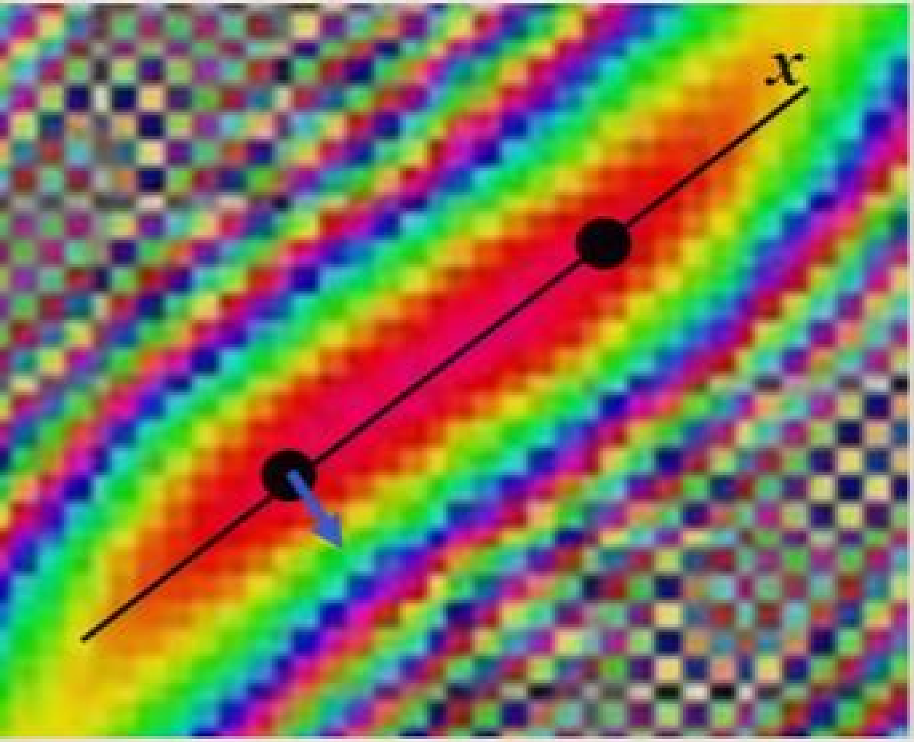
IBM Josephson junction qubit: potential landscape



--Double minimum evident (red streak)

--Third direction very "stiff"

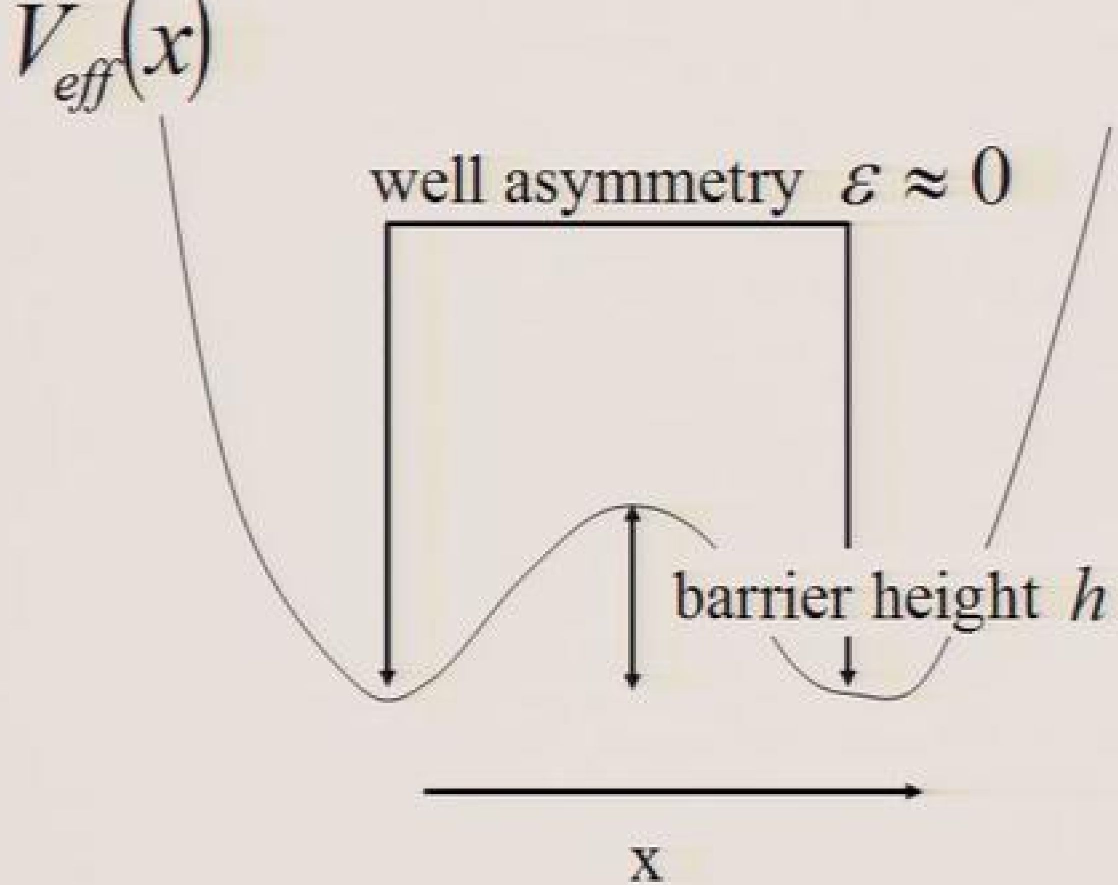
IBM Josephson junction qubit: effective 1-D potential



--treat two transverse directions (blue) as "fast" coordinates using Born-Oppenheimer

$$V_{eff}(x) = V_{line}(x) + \frac{1}{2}\hbar\omega_{trans,1} + \frac{1}{2}\hbar\omega_{trans,2}$$

IBM Josephson junction qubit: features of 1-D potential



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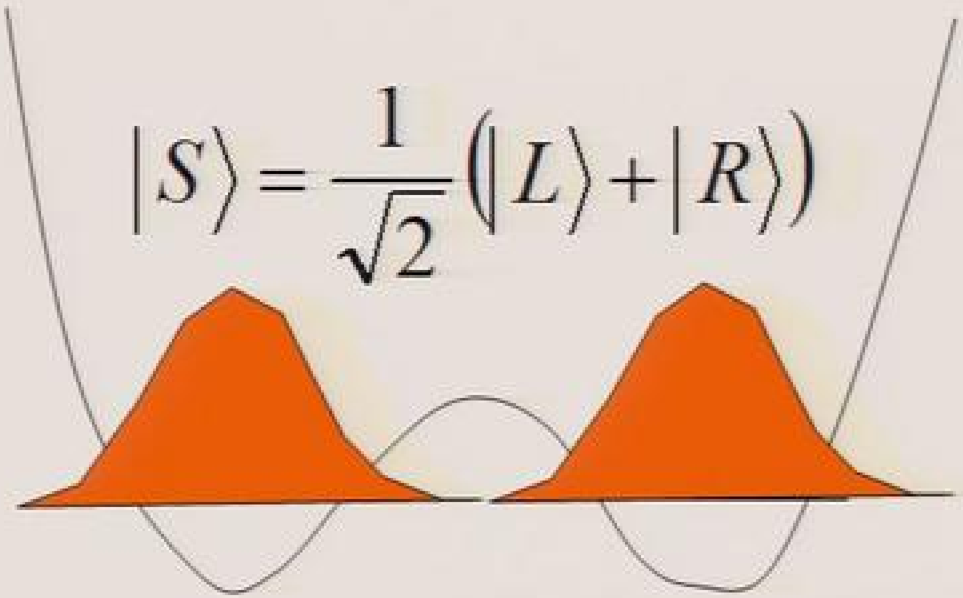
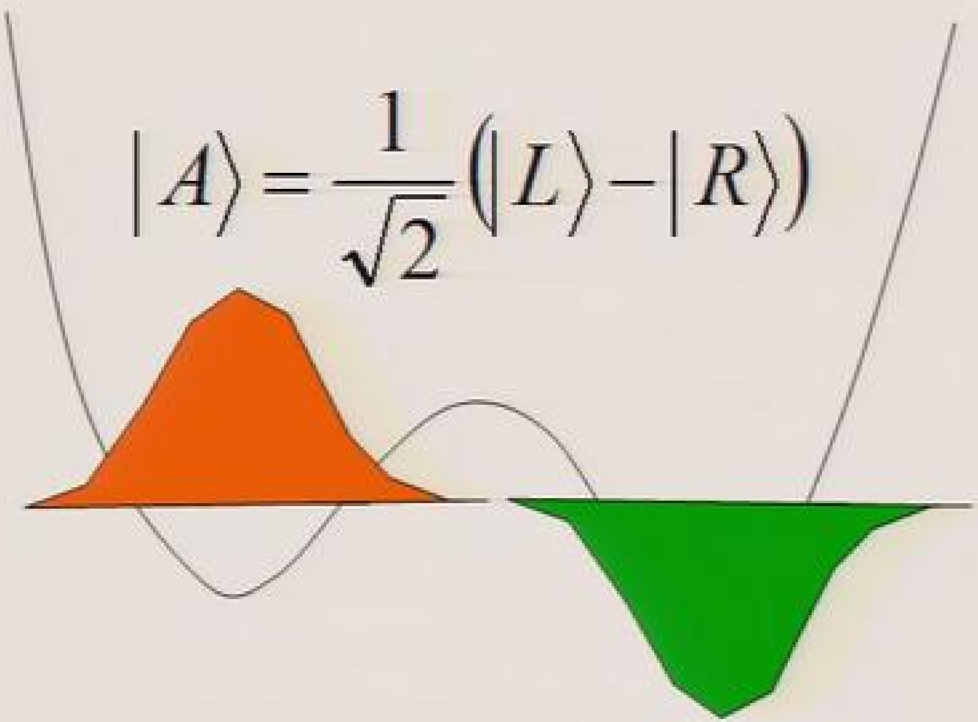


$$|S\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle + |R\rangle\right)$$

well
energy
levels –
tunnel split
into Symmetric and
Antisymmetric states

IBM Josephson junction qubit: features of 1-D potential

$$|A\rangle = \frac{1}{\sqrt{2}}\left(|L\rangle - |R\rangle\right)$$



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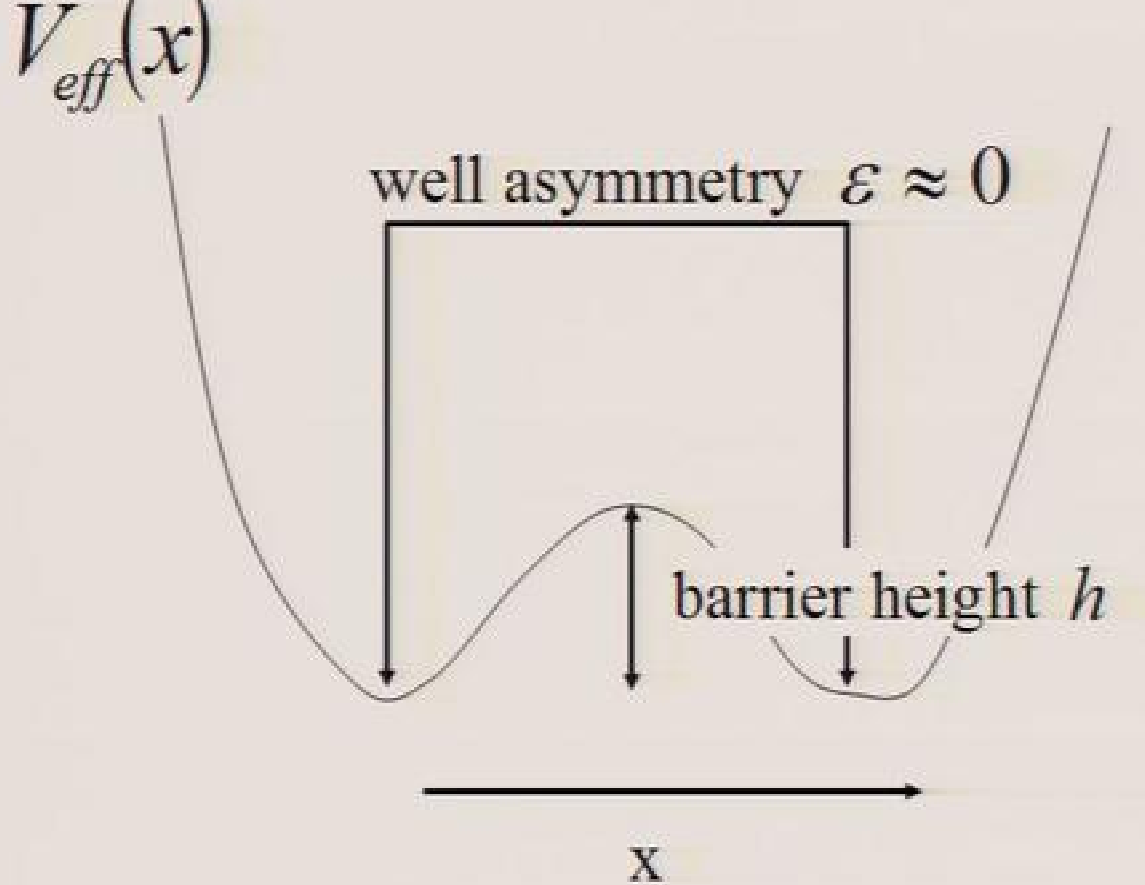
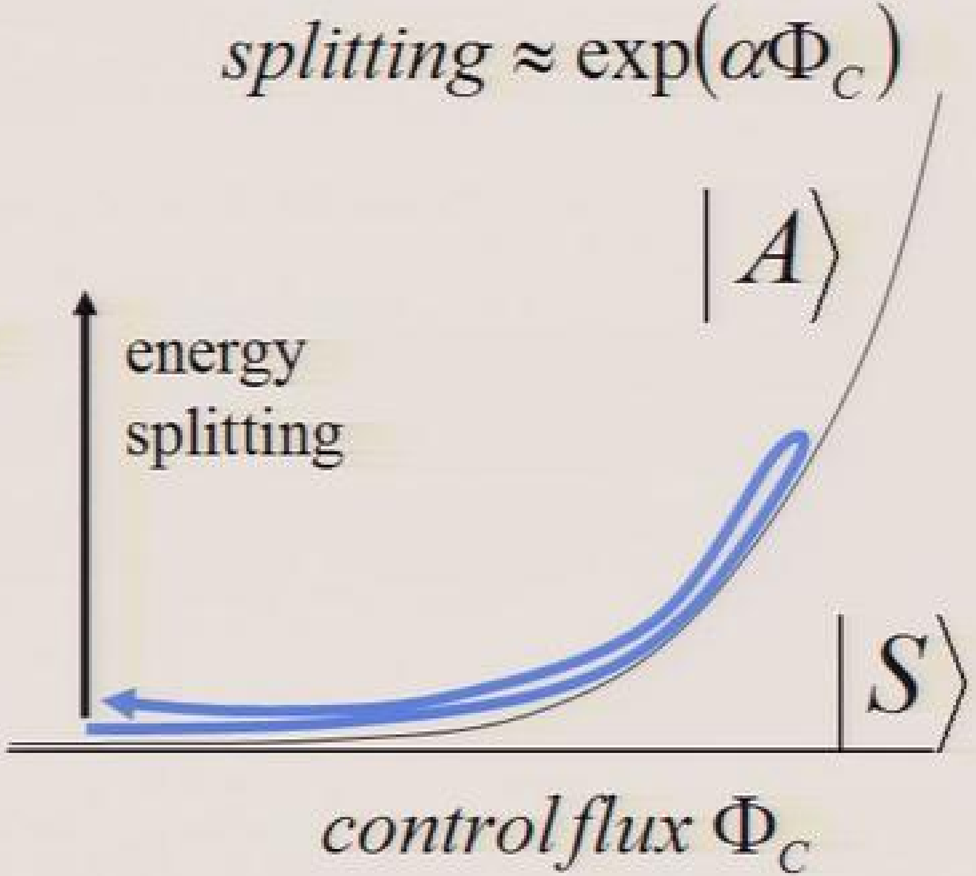


IBM Josephson junction qubit: scheme of operation:

--fix ε to be zero
--initialize qubit in state

$$|L\rangle = \frac{1}{\sqrt{2}}\left(|S\rangle + |A\rangle\right)$$

--pulse small loop flux, reducing barrier height h



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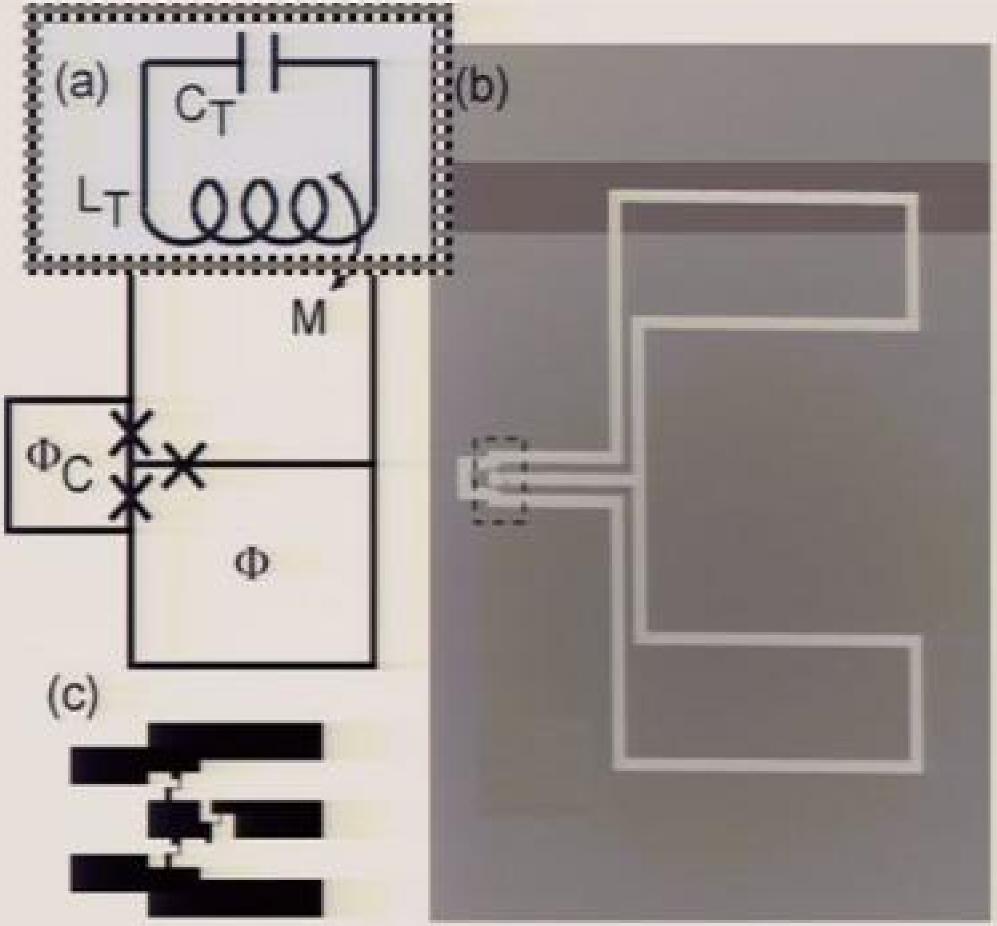

Couple qubit to harmonic oscillator (fundamental mode of superconducting transmission line). Changes the energy spectrum to:

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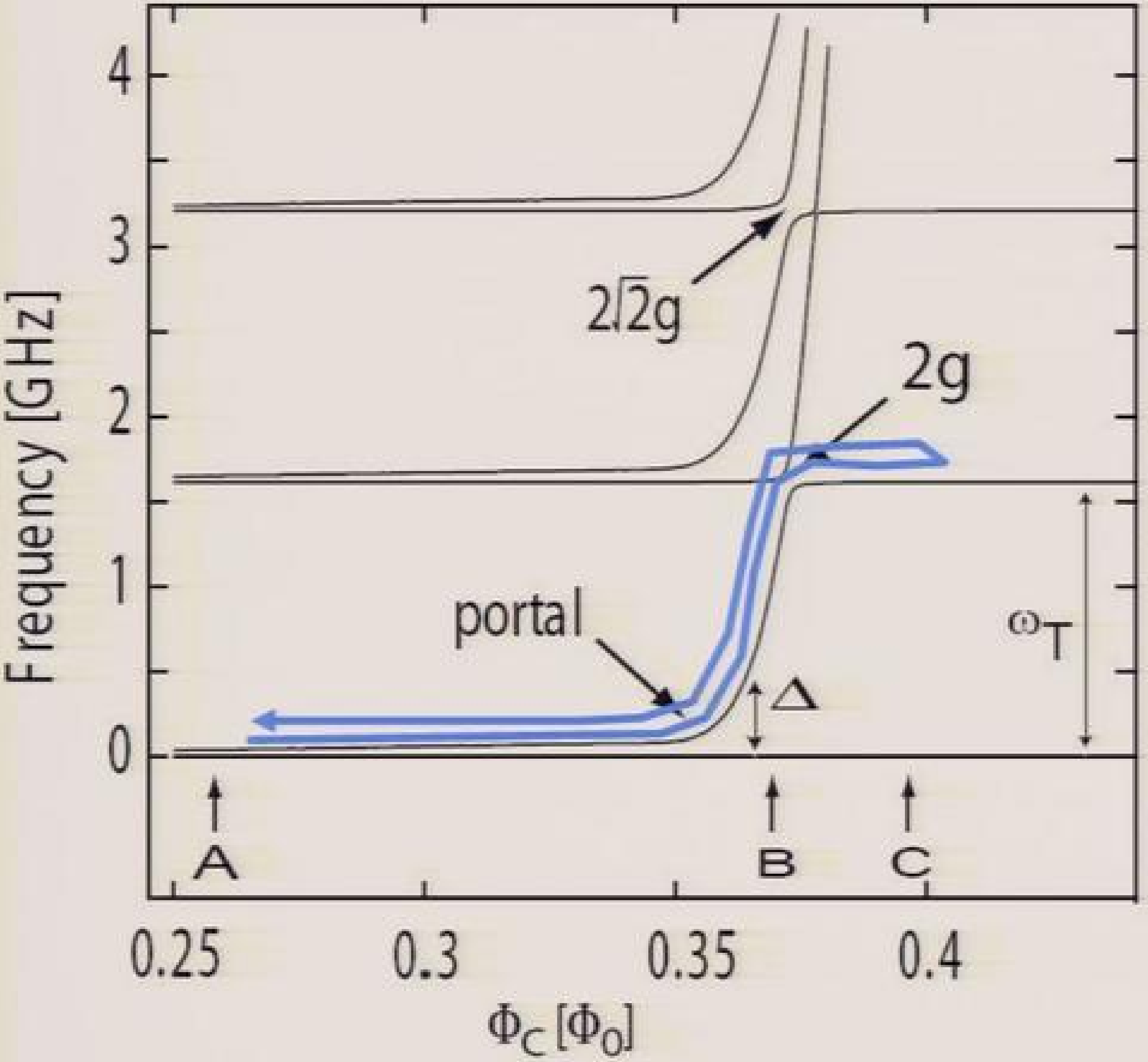
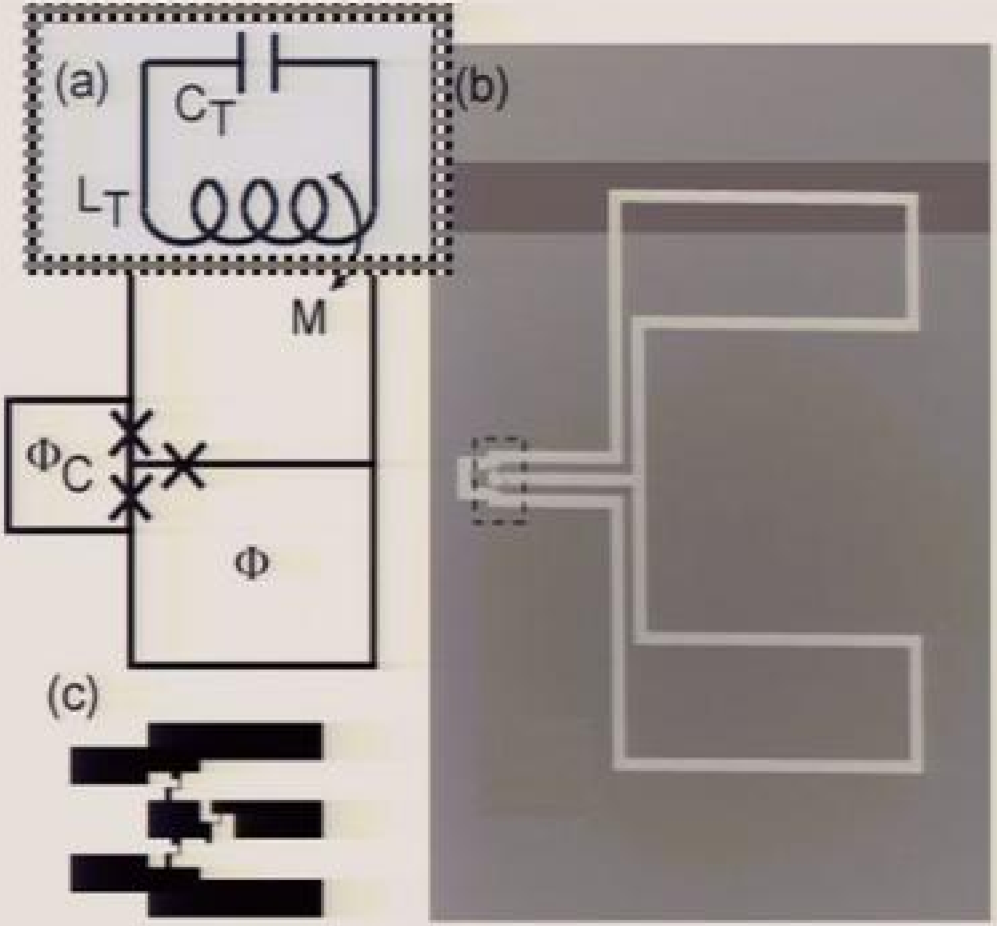


--horizonal lines in spectrum: harmonic oscillator levels (indep. of control flux)

--pulse of flux to go adiabatically past anticrossing at B, then top of pulse is in very quiet part of the spectrum

IBM Josephson junction qubit



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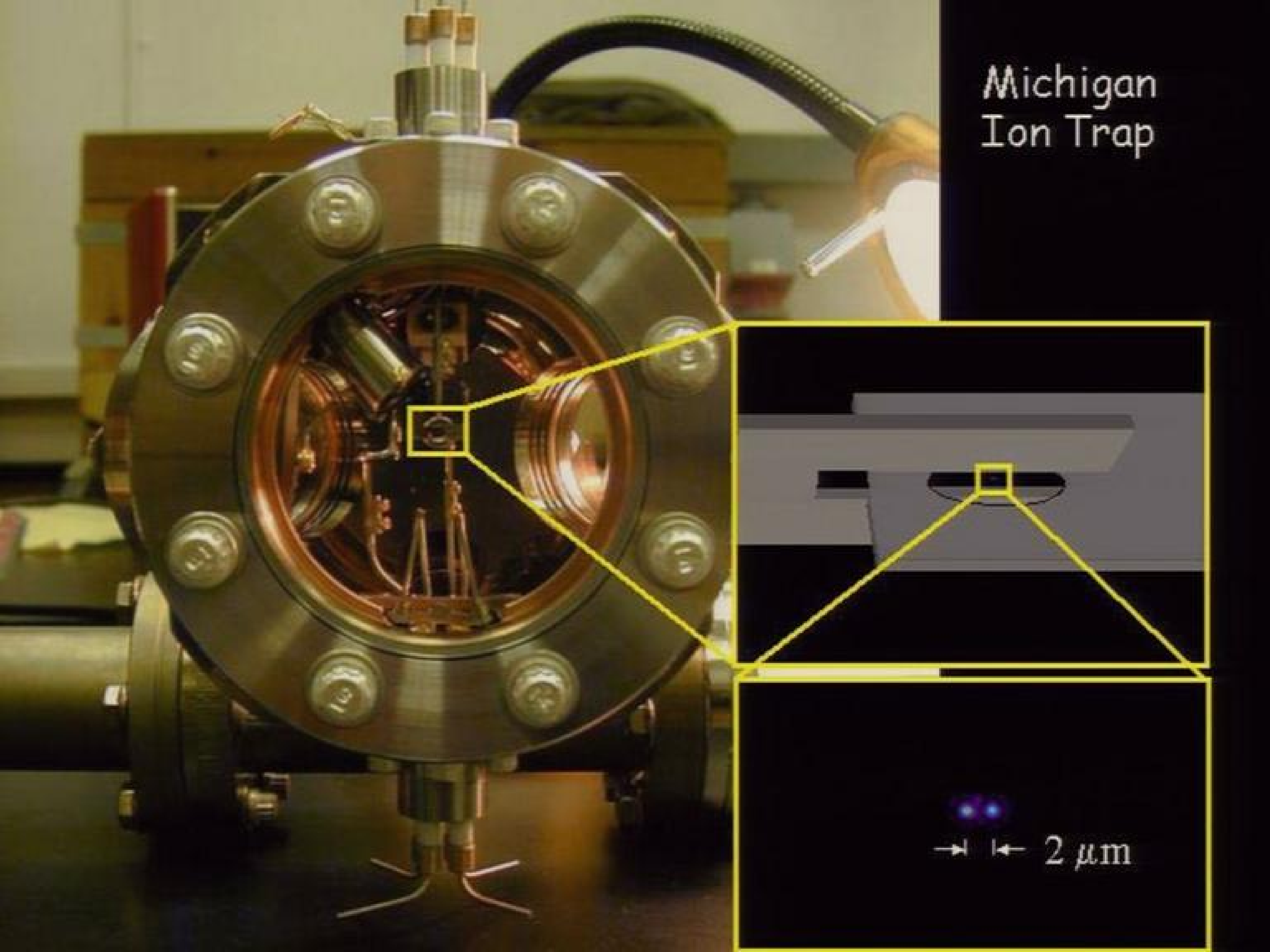
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